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Orbiting membranes in M-theory on $AdS_7 \times S^4$ background

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Abstract: We study classical solutions describing rotating and boosted membranes on $AdS_7 \times S^4$ background in M-theory. We find the dependence of the energy on the spin and R-charge of these solutions. In the flat space limit we get $E \sim S^{2/3}$, while for AdS at leading order $E - S$ grows as $S^{1/3}$. The membranes on $AdS_4 \times S^7$ background have briefly been studied as well.

Keywords: Penrose limit and pp-wave background, AdS-CFT and dS-CFT Correspondence.

Contents

1. Introduction

By now it is believed that type IIB string theory on $AdS_5 \times S^5$ with N fluxes on the S^5 is dual to four dimensional $\mathcal{N} = 4$ SU(N) SYM theory [1, 2, 3]. According to this conjecture the spectrum of single string state on $AdS_5 \times S^5$ corresponds to spectrum of single trace operators of the $\mathcal{N} = 4$ gauge theory. However, until recently, this correspondence had only been studied for supergravity modes on $AdS_5 \times S^5$ which are in one-to-one correspondence with the chiral operators of the $\mathcal{N} = 4$ gauge theory. Basically this is because formulating the appropriate sigma models and solving them for such a curved background with RR-field is not an easy work. On the other hand the massive string modes correspond to operators in long multiplets whose dimensions grow as $\lambda^{1/4}$ for large 't Hooft coupling λ. Fortunately one can get rid of these problems by considering those operators with very high bare dimension, such as operators with high R-charge, spin, etc. Practically it has been shown in [4, 10] that this procedure works for a special class of operators with high R-charge or high spin.

In fact it has been conjectured [4] that string theory on the maximally supersymmetric ten-dimensional PP-wave has a description in terms of a certain subsector of the large N four-dimensional $\mathcal{N} = 4$ SU(N) supersymmetric gauge theory at weak coupling. More precisely this subsector is parametrized by states with conformal weight Δ carrying J units of charge under the U(1) subgroup of the SU$(4)_R$ R-symmetry of the gauge theory, such that both Δ and J are parametrically large in the large 't Hooft coupling while their difference, $\Delta - J$ is finite. Then it has been possible to work out the perturbative string spectrum from gauge theory side. The idea of [4] is based on the observation that the PP-wave background with RR 4-form in type IIB string theory is maximally supersymmetric [5], solvable [6, 7] and can be understood as a certain limit (Penrose limit) of $AdS_5 \times S^5$ geometry [8, 4]. Although in the BMN consideration the background is maximally supersymmetric, it has been shown that this conjecture can also be applied for the backgrounds with less supersymmetry [9].

In an attempt to explore this idea the authors of [10] identified certain classical solutions representing highly excited string states carrying large angular momentum in the AdS_5 part of the metric with gauge theory operators with high spin S and conformal dimension Δ which is identified with the classical energy of the solution in the global AdS coordinates. An interesting observation of [10] is that the classical energy of the rotating string in AdS_5 space in the limit of $S \gg \sqrt{\lambda}$ scales as

$$\Delta - S = \frac{\sqrt{\lambda}}{\pi} \ln \frac{S}{\sqrt{\lambda}} + \cdots , \tag{1.1}$$

which looks the same as logarithmic growth of anomalous dimensions of operators with spin in the gauge theory. It has also been shown that the BMN operators [4] can also be identified with classical solutions of string in AdS_5 with angular momentum in S^5 space [10].

A generalization for the case when the string is stretched along radial direction of AdS and rotating along both AdS_5 and S^5 spaces has also been studied in [11]. This solution corresponds to those operators which have both spin and R-charge in the gauge theory side. A more general solution where the string is also stretched in an angular coordinate of S^5 has been studied in [12]. For further study in this direction see [13, 14].

The aim of this article is to generalize the classical string solution for M-theory where we would have a membrane in the AdS_7 or AdS_4 background in M-theory. Regarding the fact that the AdS/CFT correspondence has also been conjectured for AdS_7 in M-theory which says that M-theory on $AdS_7 \times S^4$ is dual to (0,2) theory, one would expect to find a classical rotating and boosted membrane solution in this background representing state with spin and R-charge in (0,2) theory. The same can be considered for $AdS_4 \times S^7$

The organization of the paper is as following. In section 2, we shall review the results of [10, 11] considering the rotating and boosted string in the $AdS_5 \times S^5$ background. In this section we will rederive their solution using the Nambu action. In section 3, we will generalize this consideration for $AdS_7 \times S^4$ where we just have a Nambu-like action. The last section is devoted to discussion and some comments. We shall also make some comments for $AdS_4 \times S^7$ case.

2. Review of closed string in $AdS_5 \times S^5$

In this section we shall review the results of [10] considering a semi-classical solution of the rotating/boosted closed string stretched along the radial coordinate of $AdS_5 \times S^5$ in type IIB string theory. A solution of rotating and boosted closed string has been studied in [11, 12].

Let us start with the supergravity solution of $AdS_5 \times S^5$ written in the global coordinates

$$\begin{aligned}
ds^2 &= R^2 \left[-dt^2 \cosh^2\rho + d\rho^2 + \sinh^2\rho d\Omega_3^2 + d\psi_1^2 + \cos^2\psi_1 \left(d\psi_2^2 + \cos^2\psi_2 d\tilde{\Omega}_3^2 \right) \right] , \\
d\Omega_3^2 &= d\beta_1^2 + \cos^2\beta_1 \left(d\beta_2^2 + \cos^2\beta_2 d\beta_3^2 \right) , \\
d\tilde{\Omega}_3^2 &= d\psi_3^2 + \cos^2\psi_3 \left(d\psi_4^2 + \cos^2\psi_4 d\psi_5^2 \right) .
\end{aligned} \tag{2.1}$$

We would like to study a solution representing a rotating closed string configuration which is stretched along the radial coordinate. In order to study this system one needs to write an action for this closed string. We note, however, that there are two different, but equivalent, ways to write the action for this string configuration; either we can use the Nambu action or the Polyakov action. Since we are going to generalize this consideration to the M-theory membrane where we do not have the Polyakov-like action, we will work with the Nambu action. This will give us an insight how to generalize the string calculations to the M-theory computations.

Now the point we should notice is the symmetries the string theory has. Let us parameterize the string worldsheet by σ and τ. Then we would have reparameterization invariance which has to be fixed. We can fix it by a parameterization such that the time coordinate of space-time, t to be equal to worldsheet time, i.e. $t = \tau$. In this gauge a closed string configuration representing a rotating string with angular velocity ω on AdS_5 space stretched along the radial coordinate is given by

$$t = \tau\,, \qquad \beta_3 = \omega\tau\,, \qquad \rho(\sigma) = \rho(\sigma + 2\pi) \tag{2.2}$$

all other coordinates are set to zero. For this solution the Nambu action,

$$I = -\frac{1}{2\pi\alpha'}\int d\sigma^2\sqrt{-\det(G_{\mu\nu}\partial_a X^\mu \partial_b X^\nu)}\,, \tag{2.3}$$

reads

$$I = -4\frac{R^2}{2\pi\alpha'}\int_0^{\rho_0} d\rho\sqrt{\dot{t}^2\cosh^2\rho - \dot{\beta}_3^2\sinh^2\rho}\,, \tag{2.4}$$

where dot represents derivative with respect to τ. For our solution (2.2) $\dot{t} = 1$, $\dot{\beta}_3 = \omega$ and $\rho_0 = \coth^{-1}(\omega)$. The factor of 4 comes from the fact that we are dealing with a folded closed string. Working with one fold string, the string can be divided to four segments. Using the periodicity condition we just need to perform the integral for one quarter of string multiplied by factor 4.

The two conserved momenta conjugate to t and β_3 are the space-time energy E and spin S. Using the above Nambu action these quantities are given by

$$\begin{aligned} E &= 4\frac{R^2}{2\pi\alpha'}\int_0^{\rho_0} d\rho\frac{\cosh^2\rho}{\sqrt{\cosh^2\rho - \omega^2\sinh^2\rho}}\,, \\ S &= 4\frac{R^2\omega}{2\pi\alpha'}\int_0^{\rho_0} d\rho\frac{\sinh^2\rho}{\sqrt{\cosh^2\rho - \omega^2\sinh^2\rho}}\,, \end{aligned} \tag{2.5}$$

From the integrals (2.5) one can proceed to compute the relation between energy and spin. To do this we can use an approximation in which the string is much shorter or much longer than the radius of curvature of AdS_5. The short and long string limits are given by the limit in which $\rho_0 \to 0$ and $\rho_0 \to \infty$, respectively.

Short strings. For large ω one finds $\rho_0 \sim 1/\omega \to 0$ and therefore the string is much shorter than the radius of curvature of AdS_5. In fact in this limit the AdS_5 space can be approximated by a flat metric near the center and therefore the calculation reduces to spinning string in the flat space. In this limit the integral (2.5) can be performed, and we find

$$E = \frac{R^2}{\omega\alpha'}\,, \qquad S = \frac{R^2}{2\omega^2\alpha'} \tag{2.6}$$

so that

$$E^2 = \frac{2R^2}{\alpha'}\, S\,, \tag{2.7}$$

which is the well-known flat space Regge trajectory.

Long strings. On the other hand for $\omega \to 1$ the length of string scales as $\rho_0 \sim \frac{1}{2}\ln\frac{2}{\omega-1} \to \infty$ and therefore the string becomes long. In this case the spin is always large compare to the radius of curvature of AdS_5 space, i.e. $S \gg R^2/\alpha'$. Setting $\rho_0 = \frac{1}{2}\ln\frac{2}{\omega-1}$ in (2.5) one can find the approximate expansion of the integrals

$$\begin{aligned} E &= \frac{R^2}{2\pi\alpha'}\left(\frac{2}{\omega-1} + \ln\frac{2}{\omega-1} + \cdots\right), \\ S &= \frac{R^2}{2\pi\alpha'}\left(\frac{2}{\omega-1} - \ln\frac{2}{\omega-1} + \cdots\right), \end{aligned} \tag{2.8}$$

which can be solved to find the dependence of energy on spin, which is

$$E - S = \frac{R^2}{\pi\alpha'}\ln\left(\frac{\alpha'}{R^2}S\right). \tag{2.9}$$

Interesting enough this looks very similar to the logarithmic growth of anomalous dimensions of operators with spin in the gauge theory.

One can also consider a string rotating in AdS_5 and in S^5 with independent angular velocity parameters ω and ν which is stretched along the radial coordinate ρ from $\rho = 0$ up to some $\rho = \rho_{max} \equiv \rho_0$. This closed string configuration is given by

$$t = \kappa\tau\,, \qquad \beta_3 = \omega\tau\,, \qquad \psi_5 = \nu\tau\,, \qquad \rho(\sigma) = \rho(\sigma + 2\pi)\,. \tag{2.10}$$

This rotating and boosted closed string configuration has been studied in [11] where the authors used the Polyakov action. Now we would like to review their solution but with the Nambu action. This could give an insight how to generalize the situation for the M-theory case. Note that in the framework where the Nambu action is used we should impose the gauge fixing condition as following

$$G_{\mu\nu}\frac{\partial X^\mu}{\partial\tau}\frac{\partial X^\nu}{\partial\sigma} = 0\,, \qquad G_{\mu\nu}\left(\frac{\partial X^\mu}{\partial\tau}\frac{\partial X^\nu}{\partial\tau} + \frac{\partial X^\mu}{\partial\sigma}\frac{\partial X^\nu}{\partial\sigma}\right) = 0\,. \tag{2.11}$$

We note that setting $t = \kappa\tau$ can not fix the diffeomorphism completely and in fact there is still a freedom to redefine the parameters which are involved in the solution (2.10). Imposing the gauge fixing condition (2.11) would fix this freedom by given a relation between them. The first relation in equation (2.11) is automatically satisfied for our string configuration (2.10), while the second one leads to the following first order equation for ρ

$$\left(\frac{d\rho}{d\sigma}\right)^2 = (\kappa^2 - \nu^2)\cosh^2\rho - (\omega^2 - \nu^2)\sinh^2\rho\,. \tag{2.12}$$

The periodicity condition on ρ is satisfied by considering a folded string. The string is folded onto itself, and the interval $0 \leq \sigma < 2\pi$ is split into four segments. The function $\rho(\sigma)$ increases from zero to its maximal value which is given by setting $\frac{d\rho}{d\sigma}(\pi/2) = 0$, i.e.

$$\cosh^2 \rho_0 = \frac{\omega^2 - \nu^2}{\kappa^2 - \nu^2} \,. \tag{2.13}$$

For the solution (2.10) the Nambu action (2.3) reads

$$I = -4\frac{R^2}{2\pi\alpha'} \int d\rho \sqrt{(\kappa^2 - \nu^2)\cosh^2 \rho - (\omega^2 - \nu^2)\sinh^2 \rho} \,. \tag{2.14}$$

Using this action one can find the energy, spin and R-charge as following

$$E = -\frac{\partial I}{\partial \kappa} \,, \qquad S = \frac{\partial I}{\partial \omega} \,, \qquad J = \frac{\partial S}{\partial \nu} \,, \tag{2.15}$$

which are

$$\begin{aligned}
E &= \frac{4R^2}{2\pi\alpha'}\kappa \int_0^{\rho_0} \frac{\cosh^2 \rho \, d\rho}{\sqrt{(\kappa^2 - \nu^2)\cosh^2 \rho - (\omega^2 - \nu^2)\sinh^2 \rho}} \,, \\
S &= \frac{4R^2}{2\pi\alpha'}\omega \int_0^{\rho_0} \frac{\sinh^2 \rho \, d\rho}{\sqrt{(\kappa^2 - \nu^2)\cosh^2 \rho - (\omega^2 - \nu^2)\sinh^2 \rho}} \,, \\
J &= \frac{4R^2}{2\pi\alpha'}\nu \int_0^{\rho_0} \frac{d\rho}{\sqrt{(\kappa^2 - \nu^2)\cosh^2 \rho - (\omega^2 - \nu^2)\sinh^2 \rho}} \,.
\end{aligned} \tag{2.16}$$

One observes that

$$E = \frac{\kappa}{\nu} J + \frac{\kappa}{\omega} S \,, \tag{2.17}$$

and the periodicity also implies an other condition on the parameters

$$2\pi = \int_0^{2\pi} d\sigma = 4 \int_0^{\rho_0} \frac{d\rho}{\sqrt{(\kappa^2 - \nu^2)\cosh^2 \rho - (\omega^2 - \nu^2)\sinh^2 \rho}} \,. \tag{2.18}$$

In particular one finds $J = \frac{R^2}{\alpha'}\nu$. This condition together with equations (2.17) and (2.16) can be used to obtain the dependence of energy E on spin and R-charge. The reader is refereed to [11] for detail of this computations.

3. Rotating and boosted membrane in M-theory AdS backgrounds

Following [10] we would like to consider the semi-classical solution of *closed membrane* in the M-theory representing states with higher angular momentum on the AdS and sphere parts which can be identified with *spin* and R-charge.

Here we shall study the boosted and rotating *closed membrane* in the $AdS_7 \times S^4$ generalizing the previously studied case for type IIB on $AdS_5 \times S^5$ background [10, 11, 12].

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Let us start with the gravity solution of $AdS_7 \times S^4$ in the global coordinates

$$\begin{aligned} l_p^{-2} dS^2 &= 4R^2\Big[- dt^2 \cosh^2\rho + d\rho^2 + \sinh^2\rho\left(d\psi_1^2 + \cos^2\psi_1\, d\psi_2^2 + \sin^2\psi_1\, d\Omega_3^2\right) + \\ &\qquad + \frac{1}{4}\left(d\alpha^2 + \cos^2\alpha\, d\theta^2 + \sin^2\alpha\left(d\beta^2 + \cos^2\beta\, d\gamma^2\right)\right)\Big], \\ d\Omega_3^2 &= d\psi_3^2 + \cos^2\psi_3\, d\psi_4^2 + \cos^2\psi_3\, \cos^2\psi_4\, d\psi_5^2\,, \end{aligned} \tag{3.1}$$

where $R^3 = \pi N$. Note that we are using a unit in which R is dimensionless.

The supersymmetric action of the M-theory supermembrane has been studied in [15]. Here we shall only consider the bosonic part of the action which can be written as following

$$I = -\frac{1}{(2\pi)^2 l_p^3}\int d\xi^3\left(\sqrt{-\det(G_{\mu\nu}\partial_a x^\mu \partial_b x^\nu)} + \frac{1}{6}\epsilon^{ijk}\partial_i x^\mu \partial_j x^\nu \partial_k x^\lambda C_{\mu\nu\lambda}\right), \tag{3.2}$$

where $(\xi_1, \xi_2, \xi_3) = (\tau, \delta, \sigma)$ are coordinates which parameterize the membrane worldvolume. x^μ, $\mu = 0, \ldots, 10$ are space-time coordinates and $C_{\mu\nu\lambda}$ is the massless M-theory three form.

We look for a soliton solution corresponding to a closed membrane configuration in the background (3.1). We note that there are different membranes which could be considered depending on in which directions their worldvolume are taken. However it turns out that not all of these configurations are supersymmetric. Here we shall only consider the supersymmetric configuration of membrane. This can be given by a membrane rotating in AdS_7 and S^4 with independent angular velocity parameters ω and ν and is also stretched along the radial coordinate and another angular coordinate of $d\Omega_5$ part of AdS_7 space. This solution is given by[1]

$$\begin{aligned} t &= \kappa\tau\,, & \psi_2 &= \sqrt{2}a\delta\,, & \psi_5 &= \sqrt{2}\omega\tau\,, \\ \rho &= \rho(\sigma)\,, & \theta &= 2\nu\tau\,, & \psi_1 &= \frac{\pi}{4}\,, \end{aligned} \tag{3.3}$$

all other coordinates are set to zero. We have also periodicity condition for ρ coordinate as $\rho(\sigma + 2\pi) = \rho(\sigma)$. Therefore we are considering a membrane configuration which is folded onto itself, and the interval $0 \le \sigma < 2\pi$ is split into four segments. The function $\rho(\sigma)$ increases from zero to its maximal value, ρ_0 which is given by $d\rho/d\sigma = 0$. We note also that, the same as AdS_5 in the previous section, we should impose the gauge fixing condition which for bosonic membrane action are given by

$$G_{\mu\nu}\frac{\partial X^\mu}{\partial\tau}\frac{\partial X^\nu}{\partial\sigma} = 0\,, \qquad G_{\mu\nu}\frac{\partial X^\mu}{\partial\tau}\frac{\partial X^\nu}{\partial\delta} = 0\,, \tag{3.4}$$

and

$$L^2 G_{\mu\nu}\frac{\partial X^\mu}{\partial\tau}\frac{\partial X^\nu}{\partial\tau} = \left(G_{\mu\nu}\frac{\partial X^\mu}{\partial\delta}\frac{\partial X^\nu}{\partial\sigma}\right)^2 - \left(G_{\mu\nu}\frac{\partial X^\mu}{\partial\sigma}\frac{\partial X^\nu}{\partial\sigma}\right)\left(G_{\lambda\gamma}\frac{\partial X^\lambda}{\partial\delta}\frac{\partial X^\gamma}{\partial\delta}\right), \tag{3.5}$$

[1] The rotating membranes in the matrix model has been studied in [16]. For supermembrane solution on PP-wave background see also [17].

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where L is a constant with dimension of length. For our solution (3.3) $L = 2R$ and moreover the constraints (3.4) are automatically satisfied while the last one, (3.5), leads to the following first order equation for ρ

$$\left(\frac{d\rho}{d\sigma}\right)^2 = \frac{(\kappa^2 - \nu^2)\cosh^2\rho - (\omega^2 - \nu^2)\sinh^2\rho}{a^2 \sinh^2\rho}, \tag{3.6}$$

which can be solved for σ. In fact from periodicity condition of ρ we find

$$2\pi = \int_0^{2\pi} d\sigma = 4a \int_0^{\rho_0} \frac{\sinh\rho \, d\rho}{\sqrt{(\kappa^2 - \nu^2)\cosh^2\rho - (\omega^2 - \nu^2)\sinh^2\rho}}, \tag{3.7}$$

and moreover we rescaled δ such that $\frac{4a}{2\pi} = 1$.

For the solution (3.3) the CS part of the membrane action (3.2) is zero and therefore the membrane action (3.2) reads

$$I = -\frac{(2R)^3}{(2\pi)^2} a \int d\tau \, d\delta \, d\rho \, \sinh\rho \sqrt{\kappa^2 \cosh^2\rho - \omega^2 \sinh^2\rho - \nu^2}\,. \tag{3.8}$$

From (2.15) we can find the energy, spin and R-charge of the membrane as following

$$\begin{aligned}
E &= \frac{4R^3}{\pi}\,\kappa \int_0^{\rho_0} \frac{\sinh\rho\,\cosh^2\rho\, d\rho}{\sqrt{(\kappa^2 - \nu^2)\cosh^2\rho - (\omega^2 - \nu^2)\sinh^2\rho}}, \\
S &= \frac{4R^3}{\pi}\,\omega \int_0^{\rho_0} \frac{\sinh^3\rho\, d\rho}{\sqrt{(\kappa^2 - \nu^2)\cosh^2\rho - (\omega^2 - \nu^2)\sinh^2\rho}}, \\
J &= \frac{4R^3}{\pi}\,\nu \int_0^{\rho_0} \frac{\sinh\rho\, d\rho}{\sqrt{(\kappa^2 - \nu^2)\cosh^2\rho - (\omega^2 - \nu^2)\sinh^2\rho}},
\end{aligned} \tag{3.9}$$

where ρ_0 is

$$\rho_0 = \coth^{-1}\sqrt{\frac{\omega^2 - \nu^2}{\kappa^2 - \nu^2}}\,. \tag{3.10}$$

The same as AdS_5 case we have

$$E = \frac{\kappa}{\nu} J + \frac{\kappa}{\omega} S\,, \tag{3.11}$$

which may be used to determine the dependence of E on S and J. The factor 4 has the same origin as one in AdS_5 in the previous section. An immediate consequence of periodicity condition is that $J = \alpha_0 \nu$ with $\alpha_0 = 4R^3/\pi$.

Now we have all ingredients to proceed to find the dependence of E on the S and J. To do this we need to perform the integrals (3.9) which are involved in our study. In fact we get

$$\begin{aligned}
S &= \alpha_0 \frac{\omega}{\sqrt{\kappa^2 - \nu^2}} \frac{1}{2\eta^{3/2}} \left[\sqrt{\eta} + (\eta - 1)\left(\tan^{-1}\sqrt{\eta} - \frac{\pi}{2}\right)\right], \\
J &= \alpha_0 \frac{\nu}{\sqrt{\kappa^2 - \nu^2}} \frac{1}{\sqrt{\eta}} \left(\frac{\pi}{2} - \tan^{-1}\sqrt{\eta}\right),
\end{aligned} \tag{3.12}$$

where $\eta^{-1} = \sinh^2\rho_0$. These equations together with $J = \alpha_0 \nu$ and $E = \kappa\alpha_0 + \frac{\kappa}{\omega} S$ are enough to find the relation between E, S and J. Following [10] one can consider the limit of *short* ($\rho_0 \to 0$ or $\eta \to \infty$), and *long* ($\rho_0 \to \infty$ or $\eta \to 0$) membranes.

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Short membranes. For $\eta \to \infty$ we get

$$\kappa^2 \approx \nu^2 + \frac{1}{\eta^2}, \qquad \omega^2 \approx \nu^2 + \frac{1}{\eta}, \tag{3.13}$$

and from (3.12) one finds

$$S \approx \frac{2}{3}\alpha_0 \frac{1}{\eta}\sqrt{\nu^2 + \frac{1}{\eta}}. \tag{3.14}$$

For the case of $\nu \ll \frac{1}{\eta}$ we can solve the above equation for η

$$\frac{1}{\eta^{3/2}} \approx \frac{3}{2}\alpha_0^{-1} S, \tag{3.15}$$

and using the relation between E and S we get

$$E = \alpha_0 \sqrt{\nu^2 + \frac{1}{\eta^2}} + \cdots \qquad \Longrightarrow \qquad E \approx \alpha_0 \frac{1}{\eta}, \tag{3.16}$$

or

$$E \approx \left(\frac{9}{\pi}\right)^{1/3} R S^{2/3}. \tag{3.17}$$

Physically, in the short membrane limit, $\rho_0 \to 0$, the membrane is not stretched much compared to the radius of curvature of AdS_7, therefore we can approximate AdS_7 by flat metric near center. In this case the calculation reduces to the rotating membrane in the flat space and in fact (3.17) is analogues to the string in flat space where we get Regge trajectory, $E \sim \sqrt{S}$.

On the other hand for $1/\eta \ll \nu \ll 1/\sqrt{\eta}$ one finds

$$E = J + \left(\frac{\pi^2}{24}\right)^{1/3} R^{-2} J S^{2/3} + \cdots, \tag{3.18}$$

while for $\nu \gg 1/\sqrt{\eta}$ we get

$$E = J + S + \frac{3}{2\pi^2} R^6 \frac{S^2}{J^3} + \cdots. \tag{3.19}$$

Long membranes. In the long membranes limit where $\eta \to 0$ one finds

$$\kappa^2 \approx \nu^2 + \frac{\pi^2}{4\eta}, \qquad \omega^2 \approx \nu^2 + \frac{\pi^2}{4} + \frac{\pi^2}{4\eta}, \tag{3.20}$$

moreover

$$S \approx \frac{\alpha_0}{2}\frac{1}{\eta}\sqrt{\nu^2 + \frac{\pi^2}{4} + \frac{\pi^2}{4\eta}}. \tag{3.21}$$

Therefore the energy will be given by

$$E \approx \alpha_0 \sqrt{\nu^2 + \frac{\pi^2}{4\eta}} + \frac{\sqrt{\nu^2 + \frac{\pi^2}{4\eta}}}{\sqrt{\nu^2 + \frac{\pi^2}{4} + \frac{\pi^2}{4\eta}}} S. \tag{3.22}$$

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Note that in the long membranes limit the spin is always large compare to the radius of curvature of AdS_7 space. In this limit one can expand the above expression for energy. Using (3.21) we get the following dependence of energy on spin and R-charge

$$E - S = 3S\left[\frac{1}{2}\left(\frac{R^3}{S}\right)^{2/3} + \frac{1}{8}\left(\frac{R^3}{S}\right)^{4/3} + \cdots\right] + \\ + \frac{J}{4}\left[\frac{1}{2}\frac{J}{R^3}\left(\frac{R^3}{S}\right)^{1/3} + \frac{J}{R^3}\left(\frac{R^3}{S}\right)^{2/3} - \frac{1}{16}\left(\frac{J}{R^3}\right)^3\left(\frac{R^3}{S}\right)^{2/3} + \cdots\right]. \tag{3.23}$$

On the other hand for the limit in which the angular momentum on S^4 is very small, $\nu \to 0$, the above expansion reads

$$E - S = 3S\left[\frac{1}{2}\left(\frac{R^3}{S}\right)^{2/3} + \frac{1}{8}\left(\frac{R^3}{S}\right)^{4/3} + \cdots\right], \tag{3.24}$$

which can be thought as a perturbative expansion with the expansion parameter $\frac{R}{S^{1/3}}$. In fact in this limit a general form of the S dependence of E can be written as following

$$E = S\left[2\left(\frac{R}{S^{1/3}}\right)^2 + \frac{1}{\sqrt{1+\left(\frac{R}{S^{1/3}}\right)^2}}\right] = S\sum_{n=0}^{\infty} c_n\left(\frac{R}{S^{1/3}}\right)^{2n}, \tag{3.25}$$

where c_n is some numerical factor. This is our prediction for the relation between energy and spin. Unfortunately our knowledge about (0,2) theory which is conjectured to be dual to the M-theory in this background [1] is too little to check this expression in the (0,2) theory side.

For the case of $\frac{\pi}{2} \ll \nu \ll \frac{\pi}{2\eta}$ we get

$$E - S = 2R^2 S^{1/3} + \frac{1}{4}R^{-1}\frac{J^2}{S^{2/3}} - \frac{1}{2^6}R^{-6}\frac{J^4}{S}, \tag{3.26}$$

while for the limit of $\nu \gg \frac{\pi}{2\eta}$ one finds

$$E = J + S + 2R^4\frac{S^{2/3}}{J} - 2R^6\frac{S}{J^2}. \tag{3.27}$$

4. Conclusions

In this paper we have considered semi-classical membrane solution in the M-theory on $AdS_7 \times S^4$ background representing a membrane rotating in AdS_7 and S^4 spaces with independent angular velocity parameters. We have studied the short and long membranes limits where we have obtained the dependence of energy,E, on spin, S, and R-charge, J. We have also shown that for the case of $J \to 0$ while in the flat space limit the energy is proportional to $\sqrt{S^3}$ which is analogues to the Regge trajectory in string theory, in the long membrane limit where the effects of AdS_7 are important the $E - S$ at the leading order grows as $S^{1/3}$. In fact we have been able to write a closed form for the energy in terms of spin. This would be a prediction for the relation between energy and spin of the operators with large spin in (0,2) theory.

An other interesting limit we have considered is the limit in which a short membrane is boosted with the speed of light on S^4 space. The energy as a function of spin and R-charge is given by (3.19). Indeed this corresponds to the case where we are taking the Penrose limit of $AdS_7 \times S^4$ [4]. For comparison with the result of [4] we could, for example, consider the one loop approximation around the classical solution (3.3) with $\omega = 0$. Although in this paper we have used the Nambu action all the times, it seems using this action for the one loop approximation is problematic. This is very similar to the string case [11] where the induced metric had singularity. In the string case the authors of [11] could get rid of this problem using the Polyakov action. But since in our case we have no such an action one might wonder how to find a proper action for the fluctuations around our membrane solution. A way to do this could be to write the matrix model from membrane action. Then we can compare this matrix model with one considered in [4] (see also [19]). To do this we should first fix the light-cone gauge. In the light-cone gauge the classical membrane solution (3.3) is given in terms of x^+ and x^-, which is $x^+ = 2\nu\tau\,, \qquad x^- = 0$. Then we need to consider small fluctuations around the shrinking closed membrane

$$x^+ = 2\nu\tau + \frac{\tilde{x}^+}{R}\,, \qquad \rho = \frac{\tilde{\rho}}{R}\,, \qquad x^- = \frac{\tilde{x}^-}{R^2}\,, \qquad \zeta_i = \frac{\tilde{\zeta}_i}{R}\,, \tag{4.1}$$

where ζ_i stands for other coordinates . It can be shown that the light-cone hamiltonian (see [20]) in the quadratic approximation for bosonic membrane solution given by (4.1) in the limit of $R \to \infty$ reduces to the light-cone hamiltonian of a membrane in the PP-wave background [19].

In this paper we have considered the case in which the membranes are only stretched along the radial coordinate. But the same as [12] one could consider a more general solution where the membranes are also stretched along an angular coordinate in S^4 part.

Similarly, one could also study the classical membrane solution in $AdS_4 \times S^7$ background of M-theory. The gravity solution of $AdS_4 \times S^7$ in the global coordinate is given by

$$\begin{aligned} ds^2 &= \frac{R^2}{4}\Big[- dt^2 \cosh^2\rho + d\rho^2 + \sinh^2\rho\,\big(d\psi_1^2 + \cos^2\psi_1\, d\psi_2^2\big) + \\ &\qquad + 4\left(d\alpha^2 + \cos^2\alpha\, d\Omega_3^2 + \sin^2\alpha\, d\tilde{\Omega}_3^2\right)\Big], \\ d\Omega_3^2 &= d\beta_1^2 + \cos^2\beta_1\,(d\beta_2^2 + \cos^2\beta_2\, d\beta_3^2) \\ d\tilde{\Omega}_3^2 &= d\beta_4^2 + \cos^2\beta_4\,(d\beta_5^2 + \cos^2\beta_5\, d\beta_6^2)\,. \end{aligned} \tag{4.2}$$

For the classical membrane solution representing a boosted membrane on S^7 which is stretched along the radial coordinate and ψ_1 we will get the same results as AdS_7. This can be understood from the fact that the Penrose limits of $AdS_4 \times S^7$ and $AdS_7 \times S^4$ spaces give the same 11-dimensional PP-wave [8, 4].

Note added: after submiting the paper, we were informed by P. Sundell that the rotating membrane in $AdS_7 \times S^4$ has also been studied in [18]. In fact their solution is an special case of (3.3) in which $\kappa = 1$ and $\nu = 0$. We note, however, that for the case of $\nu = 0$ setting $\kappa = 1$ is enough to fix the gauge while for a general form one should further impose the gauge fixing conditions (3.4) and (3.5). For the similar situation in string theory see [10] and [11, 12].

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